27525(New)

B.Sc. V Semester Degree Examination, February/March- 2023

MATHEMATICS (New)

Fourier Series, Laplace Transform and Linear Transformation

Paper : 5.1

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. **Part - A: All questions are compulsory.**
2. **Part - B: Solve any Five questions.**

PART - A

I. Answer the following questions. **(10×2=20)**

1. Define Fourier series of a function $f(x)$ of period 2π.

2. Find the Fourier coefficient b_n for the function $f(x)=x^2; -\pi < x < \pi$.

3. Prove that $L[af(t)+bg(t)]=aL[f(t)]+bL[g(t)]$ where $f(t), g(t)$ an two functions and a, b are constants.

4. Evaluate $L[\cos 3t - \cosh 3t]$.

5. Evaluate $L^{-1}\left[\frac{S+b}{S^2+a^2}\right]$.

6. Verify Convolution theorem for the function $f(t)=t$ and $g(t)=e^t$.

7. Define linear transformation.

8. If $T:U \to V$ is a linear transformation then prove that $T(-\alpha)=-T(\alpha)$.

9. Find the linear transformation $T:R^2 \to R^2$ such that $T(1, 0)=(1, 1)$ and $T(0, 1)=(-1, 2)$.

10. State Rank-Nullity theorem.

[P.T.O.

PART - B

(5×12=6

II. Answer any **Five** of the following questions.

11. Obtain the Fourier series of $f(x)=e^{-ax}$; $-\pi<x<\pi$ when $f(x)$ is period 2π.

12. Find the Fourier expansion for the function $f(x)=x-x^2; -l<x<l$.

III. 13. Find the half-range Sine series of the function $f(x)=\pi-x$ in $0<x<\pi$.

14. Evaluate $L[\cosh 4t, \sin t]$.

IV. 15. If $L[f(t)]=F(s)$ then prove that $L[f'(t)]=SF(s)-f(0)$.

16. Evaluate $L^{-1}\left[\frac{S+1}{S^2-4S-5}\right]$.

V. 17. Evaluate $L^{-1}\left[\frac{S}{S^2+S-2}\right]$.

18. Using Convolution theorem evaluate $L^{-1}\left[\frac{S^2}{\left(S^2+9\right)^2}\right]$.

VI. 19. $T:V_3(R)\to V_2(R)$ is defined by $T(x, y, z)=(x+y, y+z)$ show that T is linear transformation.

20. Find the linear transformation $T:R^2\to R^2$ su that $T(1, 1)=(0, 1)$ and $T(-1, 1)=(3, 2)$.

VII. 21. Find the matrix of linear transformation $T:R^2\to R^3$ define $T(x, y)=(2y-x, y, 3y-3x)$ relative to bases $B_1=\{(1,1), (-1,1)\}$ $B_2=\{(1,1,1), (1,-1,1), (0,0,1)\}$.

22. For the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ find the corresponding linear transformation $T : R^2 \to R^2$ with respect to basis $\{(1,0), (1,1)\}$.

VIII. 23. Let T be a linear operator on V. Let $F = ((\beta_{ij}))$ and $((\gamma_{ij}))$ be the matrices of linear operator τ relative to basis B and C respectively then show that $F((a_{ij})) = ((a_{ij}))G$ when $((a_{ij}))$ is the transition matrix.

24. Prove Rank-Nullity theorem.

27526(New)

B.Sc. V Semester Degree Examination, February/March - 2023

MATHEMATICS (New)

Differential Equations

Paper : 5.2

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1. Part - A: Answer all questions.
2. Part - B: Answer any Five questions, each question carry equal marks.

PART - A

(10×2=20)

I. Answer the following questions.

1. Find the C.F. of the equation

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}.$$

2. Find the Wronskian equation for the equation $\frac{d^2y}{dx^2} + y = \sec x$.

3. Define Total differential equation and write the condition for integrability of single differential equation.

4. Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{xy^2}$.

5. Define partial differential equation with example.

6. Form the Partial Differential equation from $x^2 + y^2 = (z-c)^2 \tan^2 \alpha$.

7. Solve $p \tan x + q \tan y = \tan z$.

8. Solve $\sqrt{p} + \sqrt{q} = x + y$.

9. Find the complete integral of the equation $(px + qy - z)^2 = 1 + p^2 + q^2$.

10. Write Charpit's Auxilliary equation.

[P.T.O

PART - B

II. Answer any FIVE of the following questions. (5×12=60)

11. Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y=0$ given that $x+\frac{1}{x}$ is a solution.

12. Solve $\cos x\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-2y\cos^3 x=2\cos^5 x$ by changing the independent variable.

III. 13. Solve $x^2y_2+xy_1-y=x^2e^x,\ x>0$ by the method of variation of parameters.

14. Write a necessary and sufficient condition for the equation $a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=f(x)$ to be exact, if $\frac{d^2a_0}{dx^2}-\frac{d\,a_1}{dx}+a_2=0$, here a_0, a_1, a_2 are functions of x.

IV 15. Verify the condition for integrability and solve $3x^2dx+3y^2dy-\left(x^3+y^3+e^{2z}\right)dz=0$

16. Solve $\frac{dx}{x^2+y^2+yz}=\frac{dy}{x^2+y^2-zx}=\frac{dz}{z(x+y)}$

V. 17. Verify the condition for integrability and solve $\left(y^2+yz\right)dx+\left(xz+z^2\right)dy+\left(y^2-xy\right)dz=0$

18. Solve the simultaneous equation $\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$.

VI. 19. Form the partial differential equation by the method of elimination of arbitrary, functions f and g in $Z = \frac{1}{y}\left[f(x+ay) + g(x-ay)\right]$.

20. Solve i) $p^2 q^3 = 1$

ii) $p^2 - q^2 = 1$.

VII. 21. Solve $x^2 p^2 + y^2 q^2 = z^2$.

22. Solve $4\left(1 + Z^3\right) = 9Z^4 pq$.

VIII. 23. Find the complete integral of $px + qy = pq$ by Chárpit's method.

24. Find the complete integral of $\left(p^2 + q^2\right)y = qz$ by Charpit's method.

27527(New)

B.Sc. V Semester Degree Examination, February/March - 2023

MATHEMATICS (New)

Series Solution, Improper Integrals and Vector Analysis

Paper : 5.3

Time : 3 Hours

Maximum Marks : 80

PART - A

(10×2=20)

I. Answer the following questions.

1) Using generating function, Prove that $P_n'(1)=\frac{1}{2}n(n+1)$.

2) Prove that $J-\frac{1}{2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.

3) Prove that $\mu(n+1)=n!$

4) Using generating function show that $P_n(1)=1$.

5) Show that $\int_0^\infty e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$.

6) Show that $\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}d\theta$.

7) Find the directional derivatives of the function $f(x,y,z)=xy^2+yz^3$ at $(2,-1,1)$ in direction of $2i+j+2k$.

8) Define Divergence and curl of a vector point function.

9) If $f=2xi+3yj+4zk$ and $\oint=xy^2z^3$, find $f.\nabla\oint$ and $\nabla|f|^2$.

10) Show that curl (grad f) = 0 If $f=x^2y+2xy+z^2$.

PART - B

Answer the FIVE of the following questions. (5×1...)

II. 11) With usual notation derive the Rodrigue's formula.

12) Prove that $\int_{-1}^{1} P_m(x).P_n(x)dx = 0, \text{ if } m \neq n$.

III. 13) Show that $n P_n(x) = x P_n'(x) - P_{n-1}'(x)$

14) Show that $\int_0^1 P_{2n}(x)\,dx = 0$.

IV. 15) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$

16) For recurrence relation show that, $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

V. 17) Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\,d\theta . \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$.

18) Prove that $\beta(m,n) = \frac{\mu(m)\mu(n)}{\mu(m+n)}$.

VI. 19) Show that $\int_0^3 \frac{dx}{\sqrt{3x-x^2}} = \pi$.

20) Show that $\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx = \frac{\mu(n+1)}{(m+1)^{n+1}}$.

II. 21) If $r = |\mathbf{r}|$, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ then show that, $b.\nabla\left[a.\nabla\left(\frac{1}{r}\right)\right] = \frac{3(ar)(br)}{r^5} - \frac{ab}{r^3}$

where a and b are two constants.

22) Show that $\nabla\left[r\nabla\left(\frac{1}{r^3}\right)\right] = \frac{3}{r^4}$, where $r^2 = x^2 + y^2 + z^2$.

VIII. 23) Find the angle between the normal to the surface $xy = z^2$ at the point $(1,9,-3)$ and $(-2,-2,2)$

24) Find the unit normal vector of the point (1,-1,2) to the surface $x^2y + y^2z + z^2x = 5$.

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B.Sc. V Semester Degree Examination, February/March 2022
MATHEMATICS (New)
5.1 : Fourier Series, Laplace Transforms and Linear Transformation

Time : 3 Hours Max. Marks : 80

Instructions : *1) Part – **A** is **compulsory.***
*2) Part – **B** solve **any five** questions.*

PART – A

Answer the following questions : **(10×2=20)**

I. 1) If $f(x) = e^x, -\pi < x < \pi$, find the Fourier co-efficient a_n.

2) Find the Fourier co-efficient b_n if $f(x) = x^2$ is $x \in [-\pi, \pi]$.

3) Show that $L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$.

4) Evaluate $L\{t^3 + 3t^2 - 6t + 8\}$.

5) Evaluate $L^{-1}\left\{\frac{1}{(s-4)^3}\right\}$.

6) Verify convolution theorem for $f(t) = 1$, $g(t) = \sin t$.

7) Define linear transformation and linear map.

8) Find the linear transformation $f : R^2 \to R^2$ such that $f(1, 0) = (1, 1)$ and $f(0, 1) = (-1, 2)$.

9) Find the matrix of the linear transformation. $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (x + y, x, 3x - y)$ w.r.t. a standard bases.

10) Define rank, nullity of a linear transformation.

P.T.O.

PART – B

Answer **any five** of the following : **(12×5=60)**

II. 11) Find the Fourier series for the periodic function f(x) with period 2l. Where $f(x) = |x|. -l < x < l$.

12) Find the half range sine series of $f(x) = x, 0 \le x \le 1$.

III. 13) Obtain the Fourier series of $f(x) = \begin{cases} x & -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \frac{3\pi}{2} \end{cases}$

14) Evaluate $L^{-1}\left\{\frac{3s^2 + 16s + 26}{s\left(s^2 + 4s + 13\right)}\right\}$.

IV. 15) If $L\{f(t)\} = F(s)$ then prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n F(s)}{ds^n}$.

16) Using convolution theorem find the inverse transform of $\frac{s}{\left(s^2 + a^2\right)^2}$.

V. 17) Find the laplace transforms of the function $f(t) = \begin{cases} E & \text{for } 0 \le t \le \frac{T}{2} \\ -E & \text{for } \frac{T}{2} \le t \le T \end{cases}$ and $f(t + T) = f(t)$.

18) Find the Laplace transform of the function $\frac{2\sin 2t \sin 5t}{t}$.

VI. 19) Find the matrix of the linear transformation $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (2y - x, y, 3y - 3x)$ relative to bases $B_1 = \{(1, 1), (-1, 1)\}$ and $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.

20) Prove that if $T : u \to v$ is linear transformation then

a) $T(0) = 0'$ where 0 and 0′ are zero vectors of u and v respectively

b) $T(-\alpha) = -T(\alpha), \forall \alpha \in \mu$

c) $T(C_1 \alpha_1 + C_2\alpha_2 + \ldots + C_n\alpha_n) = C_1T(\alpha_1) + C_2T(\alpha_2) + C_3T(\alpha_3) + \ldots C_nT(\alpha_n)$.

VII. 21) Let $T : V \rightarrow W$ be a linear transformation and V be a finite demensional vector space then $\gamma(T) + \eta(T) = d(V)$.

22) If T is a linear transformation from $V_3(R)$ into $V_4(R)$ defined by $T(1, 0, 0) = (0, 1, 0, 2)$. $T(0, 1, 0) = (0, 1, 1, 0)$, $T(0, 0, 1) = (0, 1, -1, 4)$. Find the range, null space, nullity of T.

VIII. 23) For the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ find the corresponding linear transformation

$T : R^2 \rightarrow R^2$ w.r.t. the basis $\{(1, 0), (1, 1)\}$.

24) If T is mapping from $V_2(R)$ in to $V_2(R)$ defined by $T(x, y) = [x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta]$. Show that T is linear transformation.

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B.Sc. V Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 5.2 : Differential Equations (New)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Part – **A** : **All** questions are **compulsory**.*
*2) Part – **B** : Solve **any five** questions from seven questions (**Each** question carries **equal** marks).*

PART – A

I. Answer the following questions. **(10×2=20)**

1) Find the part of C.F. of the equation $(1-x)y'' + xy' - y = 0$.

2) Find the Wronskian equation for the equation $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

3) Verify the condition for integrability $(y + z)\,dx + (z + x)\,dy + (x + y)\,dz = 0$.

4) Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{xy^2}$.

5) Solve $\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$.

6) Define total differential equation.

7) From the partial differential equation from $z = (x + a)(y + b)$.

8) Solve $zp + qy = x$.

9) Solve $p = e^q$.

10) Solve $px + qy + p^2 + q^2$.

PART – B

Answer **any five** of the following questions. **(5×12=60)**

II. 11) Solve $x^2y'' + xy' - y = 2x^2$, $(x > 0)$ given that $\frac{1}{x}$ is a part of C.F.

12) Solve by the method of variable of parameters of the equation $\frac{d^2y}{dx^2} + y = \text{cosec}x$.

P.T.O.

III. 13) Show that $(1+x^2)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\sec^2 x$ is exact and solve it.

14) By changing dependent variable, solve $\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\left(1+\frac{2}{x^2}\right)y=xe^x$.

IV. 15) Verify the condition for integrability and solve $z^2dx+(z^2-2yz)\,dy+(2y^2-yz-zx)\,dz=0$.

16) Solve $\frac{dx}{x^2+y^2+yz}=\frac{dy}{x^2+y^2-zx}=\frac{dz}{z(x+y)}$.

V. 17) Verify the condition for integrability and solve $(2x^2+2xy+2xz^2+1)\,dx+dy+2zdz=0$

18) Solve $\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$.

VI. 19) Solve $(bz-cy)\frac{\partial z}{\partial x}+(cx-az)\frac{\partial z}{\partial y}=ay-bx$.

20) Solve $z(p+q)=\tan x+\tan y$.

VII. 21) Solve $z^2(p^2x^2+q^2)=1$.

22) Solve $z(p^2-q^2)=x-y$.

VIII. 23) Find the complete integral of $px+qy=pq$ by Charpits method.

24) Find the complete integral of $p^2+q^2-2px-2qy+2xy=0$ by Charpits method.

B.Sc. V Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 5.3 (New) : Series Solution, Improper Integrals and Vector Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *1) Part – **A** : **All** questions are **compulsory**.*
*2) Part – **B** : Solve **any five** from seven questions. **Each** question carries **equal** marks.*

PART – A

I. Answer the following questions. **(10×2=20)**

1) Show that $P_n(1) = 1$.

2) Using generating function for Legendre's polynomial $P_n(x)$, prove that

$P'_n(1) = \frac{1}{2}n(n+1)$.

3) Prove that $\int_0^\infty e^{-x^2}dx = \frac{1}{2}\sqrt{\pi}$.

4) Show that $\beta(p, q) = \beta(q, p)$.

5) Evaluate $\int_0^\infty e^{-t^2}\sqrt{t^3}\,dt$.

6) Show that $\beta(m, n) = 2\int_0^{\pi/2} \sin\theta^{2m-1}\cos\theta^{2n-1}d\theta$.

7) Find the maximal directional derivative of $x^2y + yz^2 - xz^3$ at $(-1, 2, 1)$.

8) Define Divergence and curl of a vector point function.

9) State the Green's theorem.

10) If $f = 2x\hat{i} + 3y\hat{j} + 4\hat{k}$ and $\oint xy^2z^3$. Find $f.\nabla\phi$ and $\nabla|f|^2$.

P.T.O.

PART – B

Answer **any five** of the following questions. **(5×12=60)**

II. 11) Prove that $\int_0^1 x\, J_n(\alpha x).\, J_n(\beta x)\, dx = \begin{cases} 0 & \text{If } \alpha = \beta \\ \frac{1}{2}[J_{n+1}(x)]^2 & \text{If } \alpha \neq \beta \end{cases}$.

12) Express $J_4(x)$ interms of $J_0(x)$ and $J_1(x)$.

III. 13) Derive the Rodrigue's formula.

14) Prove that $\int_{-1}^{1} P_m(x)\, P_n(x) dx = 0$, if $m \neq n$.

IV. 15) Show that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$, $m > 0$, $n > 0$.

16) Show that, $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$.

V. 17) Express $\int_0^1 x^3(1-x^2)^{\frac{5}{2}} dx$ interms of Beta function.

18) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

VI. 19) Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$.

20) Show that $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}} = \frac{1}{5005}$.

VII. 21) Show that f= (siny + z) i + (x cosy – z) j + (x – y)k is irrotational, find the function ϕ such that $f = \nabla\phi$.

22) If $\vec{r}$ represent the position vector of a point P, then show that,
1) $\operatorname{div}\vec{r} = 3$. 2) $\operatorname{curl}\vec{r} = 0$.

VIII. 23) Prove that $\operatorname{curl}\left(\phi f\right) = \phi\, \operatorname{curl} f + \left(\operatorname{grad}\phi\right) \times f$.

24) Find the angle between the normal to the surface $xy = z^2$ at the points (1, 9, –3) and (–2, –2, 2).

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B.Sc. V Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 5.1 : Vector Calculus and Laplace Transform (Old)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **all** the Parts.*
*2) **Non-programmable** scientific calculator may be **used**.*

PART – A

Answer **any ten** questions. **(10×2=20)**

1. Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of vector i + 2j + 2k.

2. Find the ϕ such that $\nabla\phi = y^2z^3i + 2xyz^3j + 3xy^2z^2k$.

3. If $\overline{f}$ and $\overline{g}$ are irrotational, show that $\overline{f} \times \overline{g}$ is solenoidal.

4. State Stokes theorem in vector form.

5. Find the Fourier coefficient a_0 for the function $f(x) = x - x^2$; $x = -\pi$ to π.

6. Find the Fourier coefficient a_n if $f(x) = |x|$ where $x = -\pi$ to π.

7. If $f(x) = x^2$ in $[0, \pi]$, find the Fourier coefficient b_n.

8. Evaluate L[coshat – cosat].

9. Evaluate $L[e^{2t} \sin 3t]$.

10. Verify convolution theorem for the functions f(t) = t and g(t) = t.

11. Find inverse Laplace transform of $\dfrac{s+3}{s^2+9}$.

12. State convolution theorem.

P.T.O.

PART – B

Answer **any five** questions. **(5×6=30)**

13. Find the angle between the normal to the surface $xy - z^2 = 0$ at the point (1, 9, –3) and (–2, –2, 2).

14. Prove that $\text{div}(\bar{f} \times \bar{g}) = \bar{g} \text{ curl } \bar{f} - \bar{f} \text{ curl } \bar{g}$.

15. Evaluate by using Stokes theorem $\oint_C (\sin z dx - \cos x dy + \sin y dz)$ where C is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$.

16. Obtain the Fourier series for $f(x) = e^{-x}$; $0 < x < 2\pi$.

17. Obtain the half-range sine series for $f(x) = x$; $0 < x < \pi$.

18. Find the half-range cosine series for $f(x) = 2x - 1$ in $0 < x < 1$.

PART – C

Answer **any five** questions. **(5×6=30)**

19. Evaluate $L\left[\dfrac{\cos 3t - \cos 2t}{t}\right]$.

20. Evaluate :

   i) $L[\sin^2 t]$

   ii) $L[e^{2t} \cos 3t]$.

21. Evaluate $L^{-1}\left[\dfrac{s+3}{(s^2+6s+13)^2}\right]$.

22. Evaluate $L^{-1}\left[\dfrac{1}{s(s+2)(s+3)}\right]$.

23. Solve $y'' + 2y' + 17y = 0$ using Laplace transformation given $y(0) = 0$ and $y'(0) = 12$.

24. Solve $y'' - 9y = -8e^t$ given that $y(0) = 0$ and $y'(0) = 0$.

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B.Sc. V Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 5.2 (Old) : Series Solution, Total Differential Equations and Partial Differential Equation

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **all** the questions.*
*2) Mention the question number **carefully**.*

SECTION – A

I. Answer **any ten** of the following. **(2×10=20)**

1) Show that $P_n(-x) = (-1)^n P_n(x)$ by using generating function of Legendre polynomial.

2) Write the generating function for $J_n(x)$.

3) P.T. $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, \text{Sin}x$.

4) Show that $J_0'(x) = -J_1(x)$.

5) Using Rodrigues formula for the Legendre polynomial $P_n(x)$, find $P_n(1)$.

6) Verify the condition for integrability for the $(y + z)\, dx + (z + x)\, dy + (x + y)\, dz = 0$.

7) Solve the simultaneous equations $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2yz^2}$.

8) Form the partial differential equation from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

9) Solve $p^2 + q^2 = 1$.

10) Solve Lagranges linear equation p tanx + q tany.

11) Solve $z = px + py + \log pq$.

12) Write down the Charpitz auxillary equation.

P.T.O.

SECTION – B

II. Answer **any five** of the following. **(5×6=30)**

13) Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.

14) Show that $J_{5/2}(x) = -\sqrt{\frac{2}{\pi x}}\left[\frac{(3-x)^2 \sin x}{x^2} - \frac{3\cos x}{x}\right]$.

15) S.T.

i) $\cos(x \sin\theta) = J_0 - 2\cos 2\theta\, J_2 + 2\cos 4\theta\, J_4 + \ldots$

ii) $\sin(x \sin\theta) = 2\sin\theta\, J_1 + 2\sin 3\theta\, J_3 + \ldots$

using the generating function of Bessels function $J_n(x)$.

16) Verify the condition for integrability and solve the equation $(yz + 2x)\,dx + (zx - 2z)\,dy + (xy - 2y)\,dz = 0$.

17) Verify the condition for integrability and solve the equation $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})\,dz = 0$.

18) Solve $\frac{dx}{mn(y-z)} = \frac{dy}{nl(z-x)} = \frac{dz}{lm(x-y)}$.

19) Solve $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

SECTION – C

III. Answer **any five** of the following. **(5×6=30)**

20) Form a partial differential equation by the method of elimination of arbitrary function from $z = f(x + ay) + g(x - ay)$.

21) Solve $x^2p + y^2q = z^2$ by Lagrange's method.

22) Solve (i) $pq = p + q$ (ii) $p^2q^2 = 1$.

23) Solve $p(1 + q^2) = q(z - a)$.

24) Solve $p^2 = z^2(a - q^2)$.

25) Solve $pxy + pq + qy - yz = 0$ by Charpitz method.

26) Find the complete integral of $p^2x + q^2y = z$ by Charpitz method.

B.Sc. V Semester Degree Examination, Sept./Oct. 2022
MATHEMATICS
Paper – 5.3 (b) Old : Theory of Graph – I

Time : 3 Hours   Max. Marks : 80

Instructions : *1) Answer the questions* ***Section wise****.*
2) Write question number ***correctly****.*

SECTION – A

Answer **any ten** of the following. **(10×2=20)**

1. Define multiple graph and pseudograph with example.
2. Define degree of a vertex and isolated vertex.
3. Define regular graph. Prove that if G is regular, then $\bar{G}$ is regular.
4. Define line graph with example.
5. Find all spanning subgraph of K_3.
6. If G is a graph with $\delta(G) \geq K$, then G has a path of length K.
7. Show that every disconnected graph has atleast two components.
8. Draw complete bipartite graphs $K_{1,3}$ and $K_{2,2}$.
9. Define binary matrix.
10. Find the incidence matrix of the graph G in fig. (1)

G :

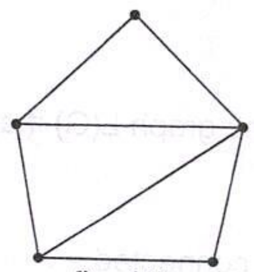



fig. (1)

11. Define adjacency matrix.

12. Define cycle matrix.

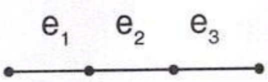
SECTION – B

Answer **any five** of the following. **(5×6=30)**

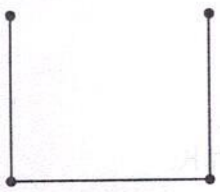
13. Show that the number of edges of an r-regular graph with p vertices in $q = \frac{pr}{2}$.

14. Draw two different self-complementary graphs with 5 vertices.

15. Find the graphs L(G), $L^2(G)$ and $T(L^2(G))$ where G is shown below

G : 

16. Find all spanning subgraphs of the following graph G.

G : 

17. Define walk, path and cycle. Prove that every u-v walk contains a u-v path.

18. Let G be a graph with p vertices and q edges. If G is bipartite, then show that $q \leq {p^2}/{4}$.

SECTION – C

Answer **any five** of the following. **(5×6=30)**

19. Show that a connected graph G is isomorphic to its line graph L(G) if and only if G is a cycle.

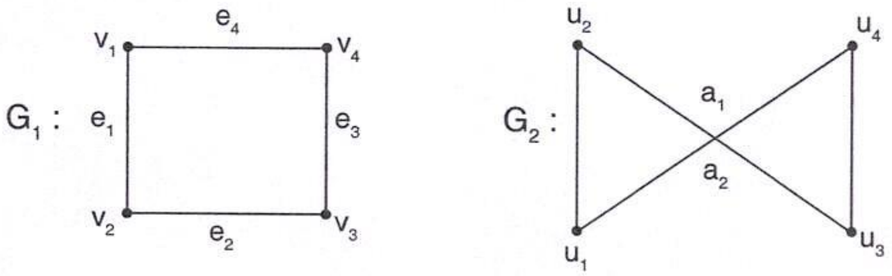
20. Prove that a graph G with P vertices and $\delta \geq \frac{p-1}{2}$ is connected.

21. Draw a self complementary graph with 5 vertices and find its incidence matrix.

22. Find the graph G whose incidence matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

23. By incidence matrix, show that the graph G_1 and G_2 shown below are isomorphic.



24. Draw K_5 and find its adjacency matrix.

27525(New)

B.Sc. V Semester Degree Examination, March - 2021

MATHEMATICS

Fourier Series, Laplace Transform, Linear Transformation

Paper : 5.1 (a)

(New)

Time : 3 Hours | Maximum Marks : 80

Instructions to Candidates : Answer All the sections.

SECTION - A

Answer the following questions. (10×2=20)

I. 1. Find the Fourier coefficient a_0 for $f(x)=x-x^2$ from $x=-\pi$ to π

2. Find the Fourier coefficient a_n if $f(x)=|x|$ where $-\pi<x<\pi$

3. Show that $L\left[e^{at}\right]=\frac{1}{s-a}$

4. Evaluate $L\left[e^{2t}\sin 4t\right]$

5. Evaluate $L^{-1}\left[\frac{s+a}{s^2+a^2}\right]$

6. Verify convolution theorem for $f(t)=t$ and $g(t)=e^t$.

7. Find the linear transformation $f:R^2\to R^2$ such that $f(1,0)=(1,1)$ and $f(0,1)=(-1,2)$

8. If $T:v_1(R)\to v_3(R)$ defined by $T(x)=(x,x^2,x^3)$ verify whether T is linear or not.

9. If $T:v_2(R)\to v_2(R)$ defined by $T(x,y)=(2x-3y\ \ x+y)$ compute the matrix to the basis $B=\{(1,2),(2,3)\}$

10. Define range and kernel of a linear transformation.

[P.T.O.

SECTION - B

Answer any **Five** of the following. (5×12=

II. 11. Obtain the Fourier series of $f(x)=\begin{cases} 1 & is \ -M_2 < x < M_2 \\ -1 & is \ M_2 < x < \frac{3\pi}{2} \end{cases}$

12. Find the half - range sine series of $f(x)=x^2$ is $0<x<\pi$.

III. 13. Find the cosine half - range series of $f(x)=x\sin x$ is $0<x<\pi$.

14. Find the laplace transform of $\left[\frac{\cos 2t-\cos 3t}{t}\right]$.

IV. 15. If $L[f(t)]=F(s)$ then prove that $L[t^n f(t)]=(-1)^n \frac{d^n}{ds^n} f(s)$

16. Evaluate $L^{-1}\left[\frac{1}{s^2-4s+13}\right]$

V. 17. Solve $y''+9y=0$ given $y(0)=0\, y'(0)=2$ using laplace transformations.

18. Solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+17y=0$ using laplace transform given y(0)=0 y'(0)=1

VI. 19. If T is a mapping from $v_2(K)$ into v_2 (R) defined by T(x,y)= $(x\cos\theta-y\sin\theta, x\sin\theta+y\cos\theta)$ show that T is a linear transform

20. If T:U → V is a linear transformation, then.

a) T (0) = 0' where 0 and 0' are zero vectors of W and V respectively.

b) $T(-\alpha)=-T(\alpha)\forall\alpha\in U$.

c) $T(c_1\alpha_1+c_2\alpha_2+......+c_n\alpha_n)=c_1T(\alpha_1)+c_2T(\alpha_2)+......+c_nT(\alpha_n)$

I. 21. If $T : v_3(R) \to v_2(R)$ defined by $T(x, y, z) = (2x + y - z,\ 3x - 2y + 4z)$ relative to basis.

$$B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$$

$$B_2 = \{(1,3), (1,4)\}$$

22. If $B = \{x_1, x_2, \ldots\ldots x_n\}$ be a basis of vector space V (f) and T be a linear transformation on V. Then prove for any vector $x \in v [T, B][x, B] = [T(x), B]$.

II. 23. State and prove Rank-Nullity theorem.

24. If T is a linear transformation from v_3 (R) into v_4 (R) define by

T (1,0,0) = (0,1,0,2)

T (0,1,0) = (0,1,1,0)

T (0,0,1) = (0,1,-1,4) then find the range, null space, rank and nullity of T.

27526(New)

B.Sc. V Semester Degree Examination, March - 2021

MATHEMATICS

Differential Equations

Paper : 5.2

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1) Part - A - All questions are compulsory.
2) Part - B - Solve any **Five** questions from **Seven** questions (Each question carries equal marks).

PART - A

I. Answer the following questions. **(10×2=20)**

1) Find the part of C.F of the equation $x^2\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}-y=e^x$.

2) Find the Wronskian equation for the equation $\frac{d^2y}{dx^2}+y=\sec x$.

3) Verify the condition for integrability $\left(2x^2+2xy+2xz^2+1\right)dx+dy+2zdz=0$.

4) Solve $\frac{dx}{z}=\frac{dy}{-z}=\frac{dz}{z^2+(x+y)^2}$.

5) Write the condition for integrability of single differential equation.

6) Define total differential equation.

7) Form the partial differential equation from $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$.

8) Solve $zp+qy=x$.

9) Solve $p+q=\sin x+\sin y$.

10) Solve $z=px+qy+p^2+q^2$.

[P.T.O

PART - B

Answer any **Five** of the following questions. (5×12=60)

II. 11) Solve $\frac{d^2y}{dx^2}+(2\cos x+\tan x)\frac{dy}{dx}+y\cos^2 x=\cos^4 x$ by change of independent variable.

12) By changing dependent variable solve $\frac{d^2y}{dx^2}-\frac{2}{x}\frac{dy}{dx}+\left(1+\frac{2}{x^2}\right)y=xe^x$.

III. 13) Solve $(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=(1-x)^2, x\neq 1$ by method of variation of parameters.

14) Show that the equation $(1+x^2)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\sec^2 x$ is exact and solve it.

IV. 15) Verify the condition for integrability and solve.

$z^2dx+(z^2-2yz)dy+(2y^2-yz-zx)dz=0$.

16) Solve $\frac{dx}{x^2+y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$.

V. 17) Verify the condition for intergrability and solve

$3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$

18) Solve $\frac{dx}{x^2+yz+z^2}=\frac{dy}{z^2+zx+x^2}=\frac{dz}{x^2+xy+y^2}$.

VI. 19) Solve $(bz-cy)\frac{\partial z}{\partial x}+(cx-az)\frac{\partial z}{\partial y}=ay-bx$.

20) Solve $z^2(p^2x^2+q^2)=1$.

VII. 21) Solve i) $p(1+q^2)=q(z-a)$

ii) $p(1+q)=2q$.

22) Solve $(p^2-q^2)z=x-y$.

VIII. 23) Find the complete integral of $px+qy=pq$ by Charpit's method.

24) Find the complete integral of $p^2x+q^2y=z$ by Charpits method.

27527(New)

B.Sc. V Semester Degree Examination, March - 2021

MATHEMATICS

Series Solution, Improper Integrals and Vector Analysis

Paper : 5.3 (a)

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

1) PART - A All questions are Compulsory.
2) PART - B Solve any **Five** questions from **Seven** questions (Each question carries equal marks).

PART - A

I. Answer the following questions. (10×2=20)

1) Using generating function for Legendre's polynomial p_n, prove that $p_n'(1)=\frac{1}{2}n(n+1)$.

2) From the recurrence relation prove that $J_n(x)=\frac{x}{2n}\left[J_{n-1}(x)+J_{n+1}(x)\right]$.

3) Prove that $\int_0^\infty e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$.

4) Prove that $\beta(m,n)=\beta(n,m)$.

5) Evaluate $\frac{\sqrt{(7)}}{2\sqrt{4}\sqrt{3}}$.

6) Using Gamma function evaluate $\int_0^\infty x^3 e^{-x}dx$.

[P.T.

7) Find the directional derivatives of the function $\phi(x, y, z) = xy^2 + yz^3$ at $(2,-1,1)$ in the direction of $2i + j + 2k$.

8) Define Divergence and curl of a vector print function.

9) If $f = 2x\hat{i} + 3y\hat{j} + 4\hat{k}$ and $\phi = xy^2z^3$. Find $f.\nabla\phi$ and $\nabla|f|^2$.

10) Show that curl(grad f) = 0 Where $f = x^2y + 2xy + z^2$.

PART - B

Answer any **Five** of the following questions. (5×12=

II. 11) With usual notation prove that $np_n(x) = x.p_n'(x) - p_{n-1}'(x)$.

12) Deduce from Rodrigue formula $\int_{-1}^{1} f(x)p_n(x)\,dx = \frac{(-1)^n}{2^n n!}\int_{-1}^{1}(x^2-1)f^{(n)}(x)$

III. 13) Show that $\int J_3(x)\,dx = -J_2(x)\frac{-2}{x}J_1(x)$

14) Prove that

(i) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

(ii) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

IV. 15) Express $J_4(x)$ in terms of J_0 and J_1.

16) Show that $\beta(m,n) = \frac{\sqrt{(m)}\sqrt{(n)}}{\sqrt{(m+n)}}$, where m>0, n>0.

V. 17) Express $\int_0^1 x^3(1-x^2)^{\frac{5}{2}}dx$ in terms of Beta function.

18) Prove that $\sqrt{(1/2)} = \sqrt{\pi}$.

I. 19) Show that $\int_0^1 \frac{x^2}{(1-x^4)^{\frac{1}{2}}}dx \int_0^1 \frac{dx}{(1+x^4)^{\frac{1}{2}}} = \frac{\pi}{4\sqrt{2}}$.

20) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$.

II.21) Show that $f = (\sin y + z)i + (x\cos y - z)j + (x - y)k$ is irrotational find the function ϕ such that $f = \nabla\phi$.

22) If $\vec{r}$ represent the position vector of a print p, then show that

(i) div r = 3.

(ii) curl r = 0.

II.23) Prove that $div(f\times g) = g\ curlf - f\ curlg$. i.e $\nabla.(f\times g) = g(\nabla\times f) - f.(\nabla\times g)$.

24) Evaluate by Green's theorem for $\oint_c [(xy + y^2)dx + x^2 dy]$ where 'C' is the closed curve of the origin bounded by $y = x$ and $y = x^2$.

11525(Old)

B.Sc. V Semester Degree Examination, March - 2021

MATHEMATICS

Laplace Transform, Vector Calculus, Fourier Series

Paper : 5.1

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates: Answer all the parts.

PART - A

Answer any **TEN** questions. **(10×2=20)**

1. Find the normal directional derivative of $(x^3 + yz)$ at the point (-3,1,-2).

2. Find ϕ such that $\nabla\phi = y^2z^3i + 2xyz^3j + 3xy^2z^2k$ given $\phi(x,y,z) = 0$ at the point.

3. Show that the vector field $f = (6xy+z^3)i+(3x^2-z)j+(3xz^2-y)k$ is irrotational.

4. State Stoke's theorem in the vector form.

5. Find the Fourier coefficient a_0 if $f(x) = x^2; x \in [-\pi, \pi]$.

6. Find the Fourier coefficient a_n if $f(x) = |x|$ where $-l < x < l$.

7. If $f(x) = x$ in $[0, \pi]$ find the Fourier coefficient b_n.

8. Evaluate : $L[\sin^3 2t]$.

9. Evaluate : $L[e^{-2t}(\sin 2t - \cos 3t)]$.

10. Verify Convolution theorem for the function $f(t) = 1$, $g(t) = \cos t$.

11. Find the inverse Laplace transform of $\frac{s+2}{s^2+4}$.

12. State Convolution theorem.

[P.T.O.

PART - B

Answer any **FIVE** questions. **(5×6=30)**

13. Find the angle between the normal to the surface $xy - z^2 = 0$ at the point $(1, 9, -3)$ & $(-2, -2, 2)$.

14. Prove that $\nabla \times (\nabla \times \overline{f}) = \nabla(\nabla \overline{f}) - \nabla^2 \overline{f}$.

15. Evaluate by using Stoke's theorem $\oint_c \sin z\, dx - \cos x\, dy + \sin y\, dz$.

16. Obtain the Fourier series of $f(x) = \begin{cases} 1 & is \quad \frac{-\pi}{2} < x < \frac{\pi}{2} \\ -1 & is \quad \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$ and $f(x + 2\pi) = f(x)$

17. Obtain the half-range sine series where $f(x) = \begin{cases} x & is \quad 0 < x < \frac{\pi}{8} \\ \frac{\pi}{4} - x & is \quad \frac{\pi}{8} < x < \frac{\pi}{4} \end{cases}$

18. Find the half-range cosine series for the function $f(x) = 2x - 1$ is $0 < x < 1$.

PART - C

Answer any **FIVE** questions. **(5×6=30)**

19. Find the Laplace transform of the following.
    a) $\sin^2 t$.
    b) $e^{2t} \cos^2 t$.

20. Evaluate: $L\left[\frac{\cos at - \cos bt}{t}\right]$

21. Express $f(t) = \begin{cases} t & 0 < t \le 2 \\ t^2 & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transform.

22. Find inverse Laplace transform of $\left[\frac{s}{(s-3)(s^2+4)}\right]$.

23. Solve $y'' + 2y' + 17y = 0$ using Laplace transform given $y(0) = 0$, $y'(0) = 12$.

24. Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ with conditions $y(0) = 0$, $y'(0) = 0$ using Laplace transform.

11527(Old)

B.Sc. V Semester Degree Examination, March - 2021

MATHEMATICS

Graph Theory - I

Paper : 5.3 (b)

(Old)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates :
1. Answer **all** sections.
2. Write the question numbers correctly.

SECTION - A

I. Answer any **Ten** of the following. **(10×2=20)**

1. Define a graph and a finite graph with example.

2. Define degree of vertex and isolated vertex.

3. Define regular graph and show that every cubic graph has an even number of vertices.

4. Draw a (5,5) graph and find its complement.

5. Prove that a (p, q) graph is a complete graph if and only if $q = \frac{p(p-1)}{2}$

6. Define subgraph and spanning subgraph.

7. Find all spanning subgraphs of K_3.

8. Define walk, path and cycle with examples.

9. Show that every disconnected graph has at least two components.

10. Draw complete bipartite graphs $K_{1,3}$ and $K_{2,2}$.

11. Find T (P_3) and its incidence Matrix.

12. Define adjacency Matrix.

[P.T.O.

SECTION - B

II. Answer any **Five** of the following. **(5×6=30)**

13. i) If G is a (p,q) graph with $V=\{v_1, v_2, ...v_p\}$ then prove that $\sum_{i=1}^{p} \deg v_i = 2q$

ii) Prove that, in any graph G, the number of vertices of odd degree is even.

14. Draw two different cubic graphs with 6 vertices and 9 edges.

15. i) Prove that every self - complementary graph has 4n or 4n+1.

ii) Prove that if G is regular, then $\overline{G}$ is regular.

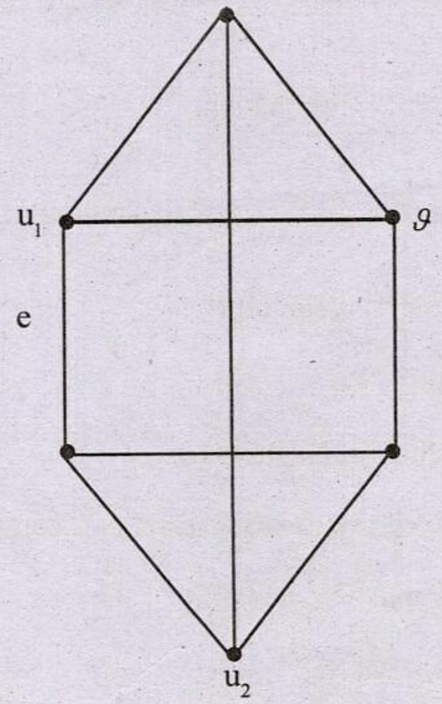
16. Draw the following subgraphs for the graph G shown below.

i) $G-v$

ii) $G-e$ and also

iii) Draw graph G+e_1 where u_1 and u_2 are not adjacent vertices of G and $e_1 = u_1 u_2$.

G :



17. Give an example of a walk, a trail, a path and a cycle.

18. Prove that a graph G with P vertices has more than $\frac{(p-1)(p-2)}{2}$ edges then G is connected.

19. Let G be a graph with P vertices and q edges. If G is bipartite, then show that $q \leq p^2/4$

SECTION - C

III. Answer any **Five** of the following. **(5×6=30)**

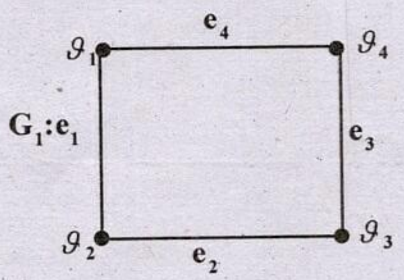
20. Find the number of vertices and edges in $K_{m,n}$

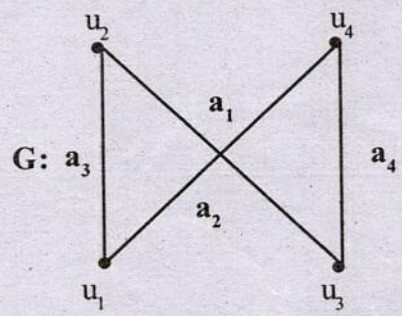
21. Prove that a graph G with p vertices and $\delta \geq \frac{p-1}{2}$ is connected.

22. Prove that a non trivial graph is bipartite if and only if all of its cycles are even.

23. Draw the graph $K_{2,3}$ and find its incidence matrix.

24. By incidence matrix, show that the graph G_1 and G_2 shown below are isomorphic.

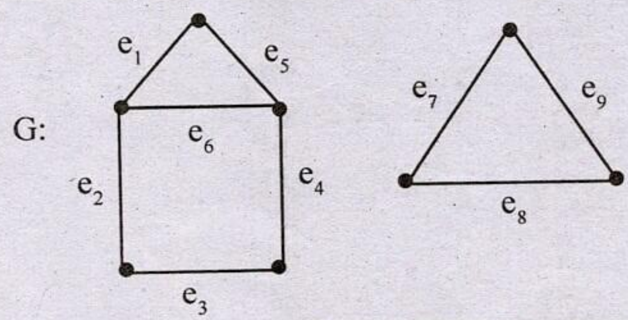




25. Find the graph G whose adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

26. Find the cycle matrix of the graph G shown below.



B.Sc. V Semester Degree Examination, Oct./Nov. - 2019

MATHEMATICS

VECTOR ANALYSIS AND LAPLACE TRANSFORMATION

PAPER-5.1

Time : 3 Hours Maximum Marks :80

Instructions to Candidates:

1) Answer **All** questions.
2) Write question numbers correctly.

SECTION -A

I. Answer any **Ten** questions: **(10×2=20)**

1. Find the directional derivative of the function $\phi(x,y,z) = xy^2 + yz^3$ at (2, -1, 1) the direction of 2i+j+2k.
2. Find the unit normal vector to the surface $3x^2+2y^2+4z^2=9$ at (1,-1,1).
3. If f = xyi+yzj+zxk, then show that $\nabla^2 f = 0$
4. Show that the vector $F = (\sin y + z)\hat{i} + [x\cos y - z]\hat{j} + (x-y)\hat{k}$ is irrational.
5. Prove that the 'Linearity property of Laplace transform.
6. Find Laplace transform of $e^{-3t}[2\cos 5t - 3\sin 5t]$.
7. Find inverse Laplace transform of $\left[\frac{s+1}{s^2+2s-8}\right]$
8. Prove that $div.curl f = 0$
9. Define Fourier series of a function.
10. Define even and odd function of x and give an example.

[P.T

11. Define Fourier series of a function of period 2L

12. Find the Fourier coefficients a_0 and a_n for the function $f(x) = 2x - x^2$ on $(0, 2)$

SECTION - B

II. Answer any **Five** of the following: (5×6=30)

13. Prove that $curl(curl\ f) = grad(div\ f) - \nabla^2 f$

14. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $curl\left(r^n.\vec{r}\right) = 0$ where $r = |\vec{r}|$.

15. Verify the green's theorem in the plane for $\oint\left[\left(xy + y^2\right)dx + x^2 dy\right]$ where c is the closed curve bounded by $y = x$ & $y = x^2$

16. Prove that $curl(f \times g) = (divg)f + (g.\nabla)f - (divf)g - (f.\nabla)g$

17. Evaluate by stoke's theorem $\oint_C yzdx + zxdy + xydz$ where C is the curve $x^2 + y^2 = 1,\ z = y^2$.

18. Find the Fourier series of $f(x) = \begin{Bmatrix} 1 & for & -\pi/2 < x < \pi/2 \\ -1 & for & \pi/2 < x < 3\pi/2 \end{Bmatrix}$ and $f(x + 2\pi) = f(x)$.

19. Express $f(x)$ as a half range sine series $f(x) = \begin{Bmatrix} \sin x & for & 0 \le x \le \pi/4 \\ \cos x & for & \pi/4 \le x \le \pi/2 \end{Bmatrix}$

SECTION - C

III. Answer any **Five** questions: (5×6=

20. Find the Laplace transform of $f(t) = \frac{kt}{p}$, $for\ 0 < t < p$ and $f(t + p) = f(t)$.

21. State and prove the convolution theorem.

22. Show that $L\left[t^n\right]=\frac{n!}{s^{n+1}}$ and hence find $L\left[t^3+3t^2+3t+1\right]$.

23. Evaluate $L^{-1}\left[\frac{s^2}{s^4+4a^2}\right]$.

24. Verify the convolution theorem for $f(t)=e^t$ and $g(t)=\cos t$

25. Show that by convolution theorem $L^{-1}\left[\frac{1}{\left(s^2+a^2\right)^2}\right]=\frac{1}{2a}[\sin at - at\cos at]$

26. Solve by Laplace transform method, $y''+2y'+5y=e^{-t}\sin t$ given $y(0)=0$ and $y'(0)=1$.

11526(New)

B.Sc. V- Semester Degree Examination, Oct/Nov. - 2019

MATHEMATICS

SERIES SOLUTION TOTAL DIFFERENTIAL EQUATIONS AND PARTIAL DIFFERENTIAL EQUATIONS

PAPER-5.2

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1. Answer **All** the sections.
2. Mention the question numbers carefully.

SECTION - A

I. Answer any **Ten** of the following : (10×2=20)

1. Show that $P_n(-x)=(-1)^n p_n(x)$ by using generating function of Legendre polynomial.
2. Using the Rodrigues formula for the legendre polynomial $p_n(x)$, find $p_1(x)$.
3. Write the generating function for $J_n(x)$.
4. Show that $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.
5. Show that $J_0^1(x)=-J_1(x)$.
6. Verify the condition of integrability of the total differential equation

   $(y+z)dx+(z+x)dy+(x+y)dz=0$.
7. Solve the simultaneous equations $\frac{dx}{y^2}=\frac{dy}{x^2}=\frac{dz}{x^2yz^2}$

[P.T.O.

8. Form the partial differential equation from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

9. Solve $p^2+q^2=1$.

10. Solve the $p^2 - q^2 = x - y$.

11. Solve Lagranges linear equation $p \tan x + q \tan y = \tan z$.

12. Find the complete integral of $z = px + qy + p^2 + q^2$.

SECTION - B

II. Answer any five of the following : (5×6

13. Prove that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

14. S.T

i) $\cos(x\sin\theta) = J_0 - 2\cos 2\theta J_2 + 2\cos 4\theta J_4 .....$

ii) $\sin(x\sin\theta) = 2\sin\theta J_1 + 2\sin 3\theta J_3 + ...$

using the generating function of Bessels function $J_0(x)$.

15. Show that $\frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)2^n p_n(x)$.

16. Verify the condition of integrability and solve the equation

$yzdx + 2zxdy - 3xydz = 0$.

17. Verify the condition for integrability and solve

$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$.

18. Solve $\frac{dx}{mn(y-z)} = \frac{dy}{nl(z-x)} = \frac{dz}{lm(x-y)}$.

19. Solve $\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$.

SECTION - C

III. Answer any **Five** of the following : (5×6=30)

20. Form the partial differential equation whose solution is

 $z=yf(x)+xg(y)$, f & g are functions.

21. Solve $x^2p+y^2q=z^2$ by Lagranges method.

22. Solve the following equations

 i. pq=p+q

 ii. Solve $p+q=Sinx+Siny$.

23. Solve $p(1+q^2)+(b-z)q=0$.

24. Solve $z(p^2-q^2)=x^2-y^2$ by using $u=\frac{2}{3}z^{\frac{3}{2}}$.

25. Solve $pxy+pq+qy-yz=0$ by charpits method.

26. Solve $p+pq-q=0$ by charpits method.

B.Sc. V Semester Degree Examination, Oct./Nov. - 2019

MATHEMATICS

GRAPH THEORY - I

PAPER- 5.3 (b)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1. Answer **All** sections.
2. Write the question numbers correctly.

SECTION - A

I. Answer any **Ten** of the following: **(10×2=20)**

1) Define a multiple graph. Give an example.

2) Define a totally disconnected graph, with an example which has 5 vertices.

3) If G is a (p,q) graph with $V=\{v_1,v_2,v_3,\ldots\ldots,v_p\}$ then prove that $\sum_{i=1}^{p}\deg v_2^1=2q$.

4) Define a complete graph K_p. Draw complete graphs K_3 and K_5.

5) Draw the graphs K_4 and $L(K_4)$.

6) What does it mean by Total graph and find $T(K_2)$.

7) Define induced subgraph and spanning subgraph.

8) Prove that $\overline{K_p}$ is a spanning subgraph of K_p.

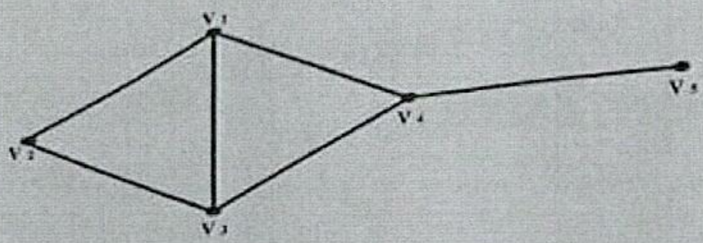
9) Define a bipartite graph. Draw all complete bipartite graphs with 5 vertices.

10) Define a walk, a path and a cycle in a graph.

11) Define an incidence matrix.

12) Write the adjacency matrix of following graph G.

G :



SECTION - B

(5×6=30)

II. Answer any five of the following:

13) i) If G be any graph with vertex set V then prove that the number of vertices of odd degree is even.

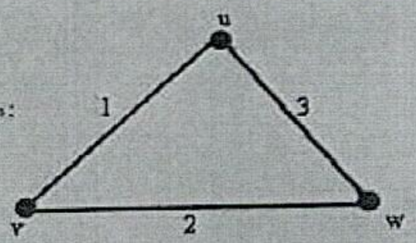
ii) Show that the number of edges of an r-regular graph with p-vertices is $q = \frac{pr}{2}$

14) Prove that for any graph G with 6 vertices G or $\overline{G}$ contains a triangle.

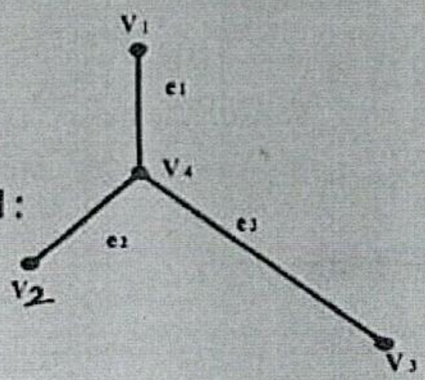
15) Show that a connected graph G is isomorphic to its line graph $L(G)$ if and only if G is a cycle.

16) Find the total graph of the following graphs

i) K_3:



ii) H :



17) Prove that every self complementary graph have $4n$ or $4n+1$ vertices.

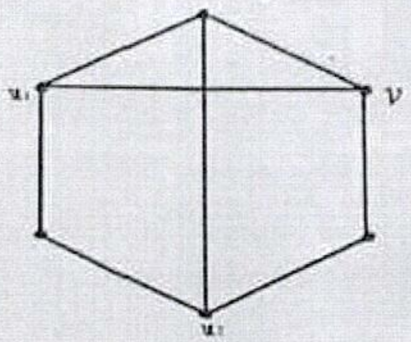
18) Draw the subgraphs of the following given graph G.

i) $G-v$

ii) $G-e$

iii) Draw $G+e$, where u_1 and u_2 are not adjacent vertices of graph G.

G:



19) Draw two different cubic graphs with 6 vertices and 9 edges.

SECTION - C

I. Answer any five questions: (5×6=30)

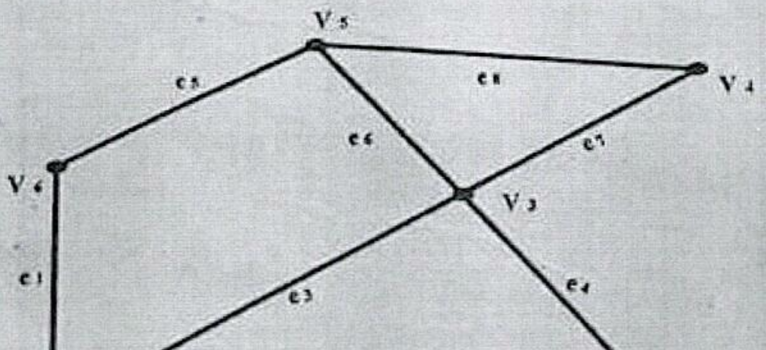
20) Prove that a graph G with p-vertices and $\delta \geq \frac{p-1}{2}$ is connected.

21) i) Draw all connected graphs with 4 vertices.

ii) If a graph G with p-vertices has more than $\frac{(p-1)(p-2)}{2}$ edges, then prove that G is connected.

22) Find the adjacency and incidence matrices of the graph G shown below.

G :



23) Draw a self complementary graph with 5 - vertices and find its incidence matri

24) Find $L(K_{1,4})$ and find its incidence matrix.

25) Find the graphs whose adjacency matrix is

i) $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

ii) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

26) Find incidence matrix A and cycle matrix C of $K_4 - e$.