31128

(NEP)

B.Sc. I Semester Degree Examination, February/March - 2023

MATHEMATICS

Algebra-1 and Calculus - 1

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

1. Part - A : ALL questions are compulsory.
2. Part -B : Answer any FIVE full questions.

PART-A

Answer the following questions. (10×1=10)

1. a) State Cayley-Hamilton theorem.

   b) Define consistent.

   c) Eigen value of a matrix is ________ iff the matrix is singular.

   d) Write the formula for length of the perpendicular from the pole to the tangent.

   e) Define pedal equation of the curve $\gamma = f(\theta)$

   f) Write the formula for radius of curvature for Cartesian curve.

   g) State the Rolle's theorem.

   h) Evaluate $\underset{x\to 0}{Lt}\dfrac{x-\sin x}{x^3}$

   i) Write the n[th] formula for a^{mx}.

   j) Define a node.

PART - B

Answer any FIVE of the following questions. (5×10=50)

a) Verify Cayley Hamilton theorem and hence find the inverse of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

b) Find the rank of the matrix A, by reducing into normal form where

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}.$$

[P.T.O.

b) Reduce $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ into Echelon form and find its rank.

3) a) Find the eigen values and the corresponding eigen vectors for the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

b) For what values of λ and μ the system

$x + y + z = 3$
$x + 2y + 2z = 6$
$x + \lambda y + 3z = \mu$ has

i) A unique solution
ii) Infinite solution
iii) No solution ?

4) a) Derive the formula for length of the perpendicular from the pole to the tangent.

b) Find the pedal equation of the curve $r = a(1 - \cos\theta)$.

5) a) Find the co-ordinates of the centre of the curvature of the curve xy at (c, c).

b) Find the envelope of the family of lines $x\cos^3\alpha + y\sin^3\alpha = a$, where α is a parameter.

6) a) State and prove Rolle's theorem.

b) Expand $\log(1 + x)$ by Maclaurin's expansion.

7) a) Evaluate :

i) $\underset{x\to 0}{\text{Lt}}\left(\frac{1}{x^2} - \frac{1}{x\tan x}\right)$

ii) $\underset{x\to \frac{\pi}{2}}{\text{Lt}}(\sec x)^{\cot x}$

b) Find the n^{th} derivative of $e^{ax}\sin(bx + c)$.

8) a) If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)y_{n-2} - (2n + 1)y_{n+1} - n^2y_n = 0$.

b) Trace the curve Astroid $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$.

31128

NEP

I Semester B.Sc. Degree Examination, March/April 2022
MATHEMATICS (New)
MATDSCT 1.1 : Algebra – I and Calculus – I

Time : 3 Hours Max. Marks : 60

Instructions : *1) Part – **A** : **All** questions are **compulsory**.*
*2) Part – **B** : Answer **any five full** questions.*

PART – A

I. Answer the following questions. **(10×1=10)**

1) a) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

b) For what values of μ and η the following system has infinite solution ? $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \mu z = \eta$.

c) Write the formula for pedal equation of a Cartesian curve.

d) What are the co-ordinates of the centre of curvature ?

e) State Lagrange's mean value theorem.

f) Evaluate $\underset{x \to 0}{\text{Lt}} \dfrac{\tan x}{x}$.

g) Find the n^{th} derivatives of e^{ax}.

h) State the Leibnitz theorem.

i) The curve $r = a(1 + \cos\theta)$ is known as

j) Write the formula for derivatives of an arc of length for $y = f(x)$.

PART – B

II. Answer **any five full** questions. **(5×10=50)**

2) a) Verify Cayley Hamilton theorem and hence find the inverse of

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

P.T.O.

b) Reduce $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 0 & 1 & -3 & -1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ into Echelon form and find its rank.

3) a) Find the eigen values and the corresponding eigen vectors for the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

b) For what values of λ and μ the system

$x + y + z = 3$

$x + 2y + 2z = 6$

$x + \lambda y + 3z = \mu$ has

i) A unique solution

ii) Infinite solution

iii) No solution ?

4) a) Derive the formula for length of the perpendicular from the pole to the tangent.

b) Find the pedal equation of the curve $r = a(1 - \cos\theta)$.

5) a) Find the co-ordinates of the centre of the curvature of the curve xy at (c, c).

b) Find the envelope of the family of lines $x\cos^3\alpha + y\sin^3\alpha = a$, where α is a parameter.

6) a) State and prove Rolle's theorem.

b) Expand $\log(1 + x)$ by Maclaurin's expansion.

7) a) Evaluate :

i) $\underset{x \to 0}{\text{Lt}} \left(\frac{1}{x^2} - \frac{1}{x \tan x} \right)$

ii) $\underset{x \to \frac{\pi}{2}}{\text{Lt}} (\sec x)^{\cot x}$

b) Find the n^{th} derivative of $e^{ax}\sin(bx + c)$.

8) a) If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)y_{n-2} - (2n + 1)y_{n+1} - n^2y_n = 0$.

b) Trace the curve Astroid $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$.

27123(New)

B.Sc. I Semester Degree Examination, March/April - 2021

MATHEMATICS

Algebra -I

Paper : 1

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates : 1. Answer all the sections.

SECTION - A

I. Answer the following questions. **(10×2=20)**

1. Find the Modulus and amplitude of 1+i

2. Simplify $\left(\frac{\sin\theta + i\cos\theta}{\cos\theta + i\sin\theta}\right)^3$

3. Find the remainder when $f(x) = x^2 - 2x + 7$ is divided by $(x-1)$

4. Find the roots of the equation $x^3 - x^2 - 5x + 6 = 0$ by Synthetic division method.

5. Using Descartes' rule of signs find the number of positive and negative roots of $x^7 + 3x^5 - 4x^4 + 7x^2 - 4x - 3 = 0$

6. Increase the root of the equation $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$ by 2.

7. Define, Symmetric and skew-symmetric matrix with example.

8. Define rank of a matrix.

9. Find the rank of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

[P.T.O.

10. Verify Cayley-Hamilton theorem for a square matrix.

$$A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

SECTION - B

Answer any **Five** of the following. **(5×12=60)**

II. 11. Simplify $\dfrac{(\cos\theta+i\sin\theta)^3(\cos\theta-i\sin\theta)^{-8}}{(\cos 4\theta+i\sin 4\theta)^5(\cos\theta+i\sin\theta)^2}$.

12. If $2\cos\theta = x+\dfrac{1}{x}$ then prove that $x^{2n}-2x^n\cos n\theta+1=0$.

III. 13. Solve $x^4+4x^3+6x^2+4x+5=0$ if 'i' is its root.

14. Solve $x^3+3x^2+3x+28=0$ by removing the second term.

IV. 15. Transform $3x^4-4x^3+4x^2-2x+1=0$ into another equation, whose leading coefficient will be unity.

16. Solve $x^3-27x+54=0$ by Cardon's method.

V. 17. Solve $x^3-3x+1=0$ by trigonometric method.

18. Solve $x^4-2x^2+8x-3=0$ by Descarte's method.

VI. 19. Find the rank of $A=\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$ by using elementary operations.

20. Find the rank of matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 2 & 3 & 4 \end{bmatrix}$ by reducing to normal form.

VII. 21. Find the inverse of following matrix by using elementary operations.

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

22. Show that the following system has unique solution and hence solve.

$$x - y - z = 3$$
$$-x - 10y + 3z = -5$$
$$2x - y + 2z = 2$$

VIII. 23. Find the eigen values and eigen vectors of the matrix. $\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$

24. Verify Cayley-Hamilton theorem for $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

0317502

27123(New)

B.Sc I Semester Degree Examination, Oct./Nov. - 2019

MATHEMATICS

ALGEBRA - I

PAPER- 1

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1. Part A - **All** questions are **compulsory**.
2. Part B solve any **Five** questions from **Seven** questions.
3. Write the question number correctly.

Part - A

I. Answer the following questions. **(10×2=20)**

1. Simplify $\left(\frac{\sin\theta + i\cos\theta}{\cos\theta + i\sin\theta}\right)^4$.

2. Find the cube root of unity.

3. Solve $x^4 - 2x^3 - 10x^2 + 6x + 45 = 0$ given that $-2+i$ is a root of the equation.

4. Show that (x+1) and (x-1) are factors of $15x^4 - 8x^3 - 11x^2 + 8x - 4$.

5. Increase the roots of the equation $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$ by 2.

6. Find the equation whose roots are reciprocals of the roots of $x^4 + px^3 + qx^2 + rx + s = 0$.

7. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.

[P.T.O.

8. Find the real values of λ for which the system has a non - trivial solution

$$2x - y + 2z = 0$$
$$3x + y - z = 0$$
$$\lambda x - 2y + z = 0$$

9. Verify the system of equations for consistent

$$x + y - 2z = 5$$
$$x - 2y + z = -2$$
$$-2x + y + z = 4$$

10. Find the eigen values of $A = \begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$

Part - B

Answer any **five** complete questions. (5×12=60)

II. 11. State De - Moivre's Theorem and prove that $\left(\frac{1+\cos\alpha + i\sin\alpha}{1+\cos\alpha - i\sin\alpha}\right)^n = \cos n\alpha + i\sin n\alpha$.

12. If $x + \frac{1}{x} = 2\cos\theta$. prove that $x^{2n} - 2x^n \cos n\theta + 1 = 0$.

III. 13. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that the sum of its real roots is zero.

14. Solve the equation $x^3 + 3x^2 + 3x + 28 = 0$ by removing the second term.

IV. 15. Transform the equation $x^4 - \frac{1}{2}x^3 + \frac{2}{9}x^2 - \frac{3}{8}x + \frac{5}{12} = 0$ into another with integral coefficients.

16. Solve the equation $x^3 - 27x + 54 = 0$ by Cardon's method.

V. 17. Solve $x^3 - 3x + 1 = 0$ by trigonometric method.

18. Solve the biguadratic equation $x^4 + 8x^3 + 9x^2 - 8x - 10 = 0$ by Descarte's method.

VI. 19. Find the rank of the matrix A by reducing it into row reduced echelon form

$$A=\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & 4 & 7 \\ -1 & -3 & 2 & -1 \\ 2 & 4 & -1 & 3 \end{bmatrix}$$

20. Find the rank of the matrix A, by reducing into its normal form where.

$$A=\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$$

VII. 21. Find the inverse of the matrix A by using elementary transformations where

$$A=\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

22. Solve completely the system of equations

$$x+2y+3z=0$$
$$y+5z=0$$
$$3x+2y+z=0$$
$$2x+3z=0$$

VIII. 23. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$

24. Verify Cayley - Hamilton theorem for the matrix A where, $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$