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B.Sc. IV Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 4 : Algebra – III, Differential Equations, Line and Multiple Integrals

Time : 3 Hours Max. Marks : 80

Instructions : *1) Part **A** : **All** questions are **compulsory**.*
*2) Part **B** : Solve **any five** questions from seven questions.*

PART – A

I. Answer the following questions. **(10×2=20)**

1) Define cyclic group and show that multiplicative group of fourth roots of unity is cyclic.

2) Define right and left coset.

3) State and prove Euler's theorem.

4) Show that the equation $(x^2 - ay)\,dx + (y^2 - ax)dy = 0$ is exact.

5) Solve $p^2 - 7p - 12 = 0$.

6) Solve $[\theta^3 - 3\theta^2 + 4]y = 0$.

7) Solve $[\theta^2 + 4]y = \sin 2x$.

8) Solve $\int_C[(3x+y)dx+(2y-x)dy]$ along line joining (0, 1) and (3, 10).

9) Evaluate $\int_0^a\int_0^b(x^2 + y^2)dydx$.

10) Solve $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$.

PART – B

Answer **any five** of the following questions. **Each** question carries **equal** marks. **(12×5=60)**

II. 11) State and prove Lagrange's theorem.

12) Show that every factor group of a cyclic group is cyclic.

P.T.O.

III. 13) State and prove quotient or factor group for the set $G|H$ of all cosets of a normal subgroup H of the group G, is a group under the binary operations defined by $Ha.Hb = Hab \forall Ha, Hb \in G|H$.

14) If $f:G \to G'$ be a homomorphism from the group $(G, \cdot)$ into the group $(G', *)$ then show that

i) $f(e) = e'$ where e and e′ are identity elements of the group G and G′ respectively.

ii) $f(a^{-1}) = [f(a)]^{-1} \ \forall a \in G$.

IV. 15) Solve the Bernoulli equation $\frac{dy}{dx} + 2y \tan x = y^2$.

16) Verify the exact equation and solve $(12x + 5y - 9)dx + (5x + 2y - 4)dy = 0$.

V. 17) Solve $y = 2px + y^2p^3$.

18) Find general and singular solution of $x^2(y - px) = yp^2$ by using the substitution $x^2 = u$ and $y^2 = v$.

VI. 19) Solve $[\theta^2 + 2\theta + 1]y = 2x + 3x^2$.

20) Solve $(x^2\theta^2 + x\theta + 9)y = 3x^2 + \sin(3 \log x)$.

VII. 21) Evaluate $\int_C [(x^2 - y)dx + (y^2 + x)\, dy]$ where C is the curve given by $x = t,\ y = t^2 + 1,\ 0 \le t \le 1$.

22) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{(a^2 - x^2 - y^2)}\, dx\, dy$.

VIII. 23) Evaluate $\iint_A \sqrt{4x^2 - y^2}\, dx\, dy$ where A is the area bounded by the lines $y = 0$, $y = x$ and $x = 1$.

24) Change the order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$.

B.Sc. IV Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 4.1 : Algebra – III (Old)

Time : 3 Hours Max. Marks : 60

Instructions : *1) Answer* ***all*** *the Sections.*
2) Write the question number ***correctly****.*

SECTION – A

Answer **any ten** of the following. **(10×2=20)**

1. Define left and right coset.
2. Define quotient or factor group.
3. State and prove Euler's theorem.
4. Find the order of the elements of the multiplicative group $G=\{1,-1,i,-i\}$ of fourth roots of unity.
5. Show that every quotient group of an abelian group is abelian.
6. Define homomorphism and Kernel of homomorphism.
7. In a ring R, prove that $a.0=0.a=0 \ \forall a \in R$, where '0' being the additive identity.
8. Define vector space over a field F.
9. Prove that intersection of any two subspaces of a vector space V over a field F is also a subspace.
10. Show that the subset $W = \{(x_1, x_2, x_3 / x_1^2 + x_2^2 + x_3^2 \leq 0)$ of $V_3(R)$ is a subspace of $V_3(R)$.
11. Is the set $\{(1,1,-1)\ (2,-3,5)\ (-2,1,4)\}$ is linearly independent.
12. Define range and Kernel of a linear transformation.

SECTION – B

Answer **any three** of the following. **(3×5=15)**

13. State and prove Lagrange's theorem.
14. State and prove fundamental theorem of homomorphism for groups.
15. Let G be a group a and H be a normal subgroup of G, then prove that G/H is a homomorphism image of G with H as its Kernel.
16. State and prove Cayley's theorem.

17. If $f : G \to G'$ be a homomorphism from the group $(G, \cdot)$ into the group $(G', *)$ show that

    i) $f(e)=e'$ where e and e' are identity elements of the group G and G' respectively.

    ii) $f(a^{-1})=[f(a)]^{-1} \forall a \in G$.

SECTION – C

Answer **any two** of the following. **(2×5=10)**

18. Prove that every integral domain is a field.

19. Define ring and show that the subset s of a ring $(R,+,\cdot)$ is a subring of R if and only if

    i) $\forall a,b \in s, a+b \in s$

    ii) $\forall a,b \in s, a.b \in s$.

20. Prove that a ring is without divisors if and only if the cancellation law holds.

21. Define subring and prove that intersection of any two subrings of a ring is again a subring.

SECTION – D

Answer **any three** of the following. **(3×5=15)**

22. Prove that a non empty subset W of a vector space V over a field F, is a subspace of V, if and only if

    i) $\alpha, \beta \in W \Rightarrow \alpha+\beta \in W$

    ii) $C \in F, \alpha \in W \Rightarrow C.\alpha \in W$.

23. Let V be a vector space over a field F then show that every nonempty subset of a linearly independent set of a vectors of V is linearly independent.

24. Define basis and dimension of a vector space V over a field F. Show that the set $B = \{(1,1,0),(1,0,1),(0,1,1)\}$ is a basis of the vector space $V_3(R)$.

25. Find the linear transformation $f : V_2(R) \to V_2(R)$ such that $f(1,1) = (0,1)$ and $f(-1,1) = (3,2)$.

26. Find the range, null space, rank and nullity of the linear transformation $T : V_3(R) \to V_2(R)$. defined by $T(x,y,z) = (y-x, y-z)$ and also verify rank nullity theorem.

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B.Sc. IV Semester Degree Examination, Sept./Oct. 2022
MATHEMATICS
Paper – 4.2 : Differential Equations (Old)

Time : 3 Hours Max. Marks : 60

Instructions : *1) Answer **all** the questions Section wise.*
*2) **Mention** the question numbers **correctly**.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Define order and degree of a differential equation and give an example.
2. Solve $(1 + x^2)\, dy + (1 + y^2)\, dx = 0$.
3. Solve the linear differential equation $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$.
4. Show that the equation $(4x + 3y + 1)\, dx + (3x + 2y + 1)dy = 0$ is exact.
5. Solve $p^2 - 5p - 6 = 0$.
6. Solve $[D^3 - 13D + 12]y = 0$.
7. Solve $(D^2 + 2D + 1)y = 2e^{2x}$.
8. Solve $\frac{1}{D^2 + a^2} \cos ax$.
9. Reduce the equation $[4x^2D^2 + 4xD - 1]y = 4x^2$ to linear differential equation with constant coefficients and hence find C.F.
10. Find a part of C.F. of the equation $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = e^{2x} \sin x$.
11. Show that the equation $(2x^2 + 3x)y'' + (6x + 3)y' + 2y = (x + 1)e^x$ is exact.
12. Find the Wronskian W for the equation $\frac{d^2y}{dx^2} + y = \mathrm{cosec}\,x$.

P.T.O.

SECTION – B

Answer **any three** of the following : **(3×5=15)**

13. Solve $\frac{dy}{dx} = (3x + 2y + 4)^2$.

14. Solve the Bernoulli equation $x\frac{dy}{dx} + y = y^2 \log x$.

15. Solve y(2xy + 1) dx – xdy = 0 by finding integrating factor.

16. Solve $y = x + 2\tan^{-1}p$.

17. Find general and singular solution $x^2(y - px) = yp^2$ by using the substitution $x^2 = u$ and $y^2 = v$.

SECTION – C

Answer **any three** of the following : **(3×5=15)**

18. Solve $(D^3 + 1)\, y = 3 + e^{-x} + 5e^{2x}$.
19. Solve $(D^2 - 2D + 5)y = \sin 3x$.
20. Solve $(D^2 - 1)y = 2 + 5x$.
21. Solve $(D^2 + 4)y = \sin^2 x$.
22. Solve the simultaneous equation $D^2x - 3x - y = e^t$ and $Dy - 2x = 0$.

SECTION – D

Answer **any two** of the following : **(2×5=10)**

23. Solve $x^2y'' + xy' - y = 2x^2$ $(x > 0)$ given that $y = \frac{1}{x}$ is a part of C.F.

24. Solve $(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} + y = 0$ using the transformation $z = \tan^{-1}x$ by change of independent variable.

25. Solve $\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$ by the method of variation of parameters.

26. Solve by changing the dependent variable $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} - (a^2 + 1)y = e^x \sec x$.

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B.Sc. IV Semester Degree Examination, April/May - 2019

MATHEMATICS

Algebra-IV

Paper - 4.1

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

1. Answer all the sections.
2. Write the question number correctly.

SECTION-A

I. Answer any **TEN** of the following questions. (10×2=20)

1. Define left and right coset.
2. Prove that Every cyclic group is abelian.
3. Define normal sub group of a group.
4. Define homomorphism and kernal of homomorphism.
5. If $f=\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 2 & 4\end{pmatrix}$ and $g=\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 2 & 3\end{pmatrix}$ find fog and gof.
6. Define a ring and give an example.
7. Define integral domain and division ring.
8. Show that on ring $(R,+,*)$

   i) $\forall a,b \in R, a(-b)=(-a)b=-(a.b)$

   ii) $\forall a,b \in R,(-a)(-b)=ab$
9. Define vector space over field F
10. Define linear transformation.
11. Define range and kernel of linear transformation.
12. State Rank Nullity theorem.

[P.T.O.

SECTION-B

II. Answer any **THREE** of the following questions. (3×5=15)

13. If 'a' is a generator of a cyclic group G, then show that $O(a)=O(G)$
14. State and prove Lagrange theorem.
15. Show that every subgroup of cyclic group is cyclic.
16. Show that a subgroup of G is normal iff $gHg^{-1}=H, \forall g \in G$
17. State and prove Quotient factor or group for the set G/H of all cosets under the binary operation defined by $Ha.Hb=Hab \ \forall Ha, Hb \in G/H$

SECTION-C

III. Answer any **TWO** of the following questions. (2×5=10)

18. Define subring and prove that intersection of any two subrings of a ring is again a subring.
19. Prove that every finite integral domain is a field.
20. A non empty subset 'S' of a ring R is a subring of 'R' iff (i) $S+(-S)=S,\ SS\subseteq S$
21. Show that the set $S=\left\{\begin{pmatrix} a & o \\ b & c \end{pmatrix}: a,b,c \in Z\right\}$ is a subring of the ring M_2 (z) for all 2×2 matrices over the set of integers.

SECTION-D

IV. Answer any **THREE** of the following questions. (3×5=15)

22. The Union of two subspace of a vector space V over a field F is a subspace iff one is contained in the other.
23. Let S be a non empty subset of a vector space $V\{F\}$ then
    i) L [S] is a subspace of V
    ii) $S\subseteq L(S)$ & L[S] is the smallest subspace of V containing 'S'.
24. Show that the set $B=\{(1,1,0),(1,0,1),(0,1,1)\}$ is a basis of the vector space V_3(R).
25. Find the matrix of the linear transformation $T:V_2(R)\to V_3(R)$ define by $T(x,y)=(2y-x,y,3y-3x)$
26. Find the range, nulspace, rank and nullity of the linear transformation $T:V_3(R)\to V_2(R)$ defined by $T(x,y,z)=(y-x,y-z)$ and also verify Rank nullity theorem.

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B.Sc. IV Semester Degree Examination, April/May - 2019

MATHEMATICS

Differential Equations - I

Paper - 4.2

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

1. Answer the questions section wise.
2. Mention the question numbers correctly.

SECTION-A

Answer any **TEN** of the following. (10×2=20)

1. Define order and degree of a differential equation and give an example.
2. Solve $\log\left(\frac{dy}{dx}\right) = ax + by$
3. Solve the linear differential equation $x\frac{dy}{dx}\log x + y = 2\log x$
4. Solve $p^2 - 7p + 12 = 0$
5. Find the general solution of $\left(D^3 - 2D^2 + 4D - 8\right)y = 0$
6. Solve $\left(D^2 + D - 6\right)y = x$
7. Solve $\left(D - 1\right)y = \cos 2x$
8. Reduce the equation to linear differential equation with constant co-efficient and hence find the C F of $x^2y_2 + xy_1 - 4y = x^2$
9. Solve $\frac{dx}{dt} + \omega y = 0$ and $\frac{dy}{dt} - \omega x = 0$
10. Find a part of C.F of the equation $(3 - x)y_2 - (9 - 4x)y_1 + (6 - 3x)y = 0$

[P.T.O

11. Find the Wranskian W for the equation $x^2y_2 + xy_1 - y = x^2e^x$

12. Show that the equation $\left(1+x^2\right)y_2 + 4xy_1 + 2y = \sec^2 x$ is exact.

SECTION-B

Answer any THREE of the following. (3×5=15)

13. Explain the method of solving the linear differential equation $\frac{dy}{dx} + Py = Q$

14. Show that $\left(2+2x^2\sqrt{y}\right)ydx + \left(x^2\sqrt{y}+2\right)xdy = 0$ is exact and hence solve it.

15. Solve $y + px = p^2x^4$

16. Solve $y = 3px + 6p^2y^2$

17. Solve $e^{3x}(p-1) + p^3e^{2y} = 0$ using the transformations $e^x = u$ and $e^y = v$.

SECTION-C

Answer any THREE of the following. (3×5=15)

18. Obtain the general solution of $\left(D^4 + 2D^3 - 3D^2\right)y = 3e^{2x} + 4\sin x$.

19. Find the general solution of $\left(D^3 - 7D - 6\right)y = e^{2x}(1+x)$

20. Solve $\left(D^2+9\right)y = \cos 2x . \cos x$.

21. Solve $x^2y_2 - 2y = x^2 + \frac{1}{x}$

22. Obtain the general solution of $\frac{dx}{dt} + 2y = -\sin t$ and $\frac{dy}{dt} - 2x = \cos t$

SECTION-D

Answer any **TWO** of the following. **(2×5=10M)**

23. Solve $\left(D^2+\frac{3}{x}D+\frac{a^2}{x^4}\right)y=0$ by the method of changing the independent variable.

24. By changing the dependent variable solve $y_2-\frac{2}{x}y_1+\left(1+\frac{2}{x^2}\right)y=xe^x$

25. Solve $(1-x)y_2+xy_1-y=(1-x)^2$, $x>1$ by the method of variation of parameters.

26. Show that $x^2(1+x)y_2+2x(2+3x)y_1+2(1+3x)y=0$ is exact and hence solve it.