27223

B.Sc. II Semester Degree Examination, September/October 2023
MATHEMATICS
Calculus – I (New)

Time : 3 Hours   Max. Marks : 80

Instructions *: 1) Part – **A** : **All** questions are **compulsory**.*
*2) Part – **B** : Solve **any five** from seven questions.*
*(**Each** question carries **equal** marks)*

PART – A

I. Answer **all** the following questions. **(10×2=20)**

1) Verify the Rolle's theorem for the function $f(x) = e^x \sin x$ in $[0, \pi]$.

2) Verify Lagrange's mean value theorem for $f(x) = x^2 - 3x - 2$ in $[-2, -3]$.

3) State the Taylor's theorem.

4) Evaluate $\lim\limits_{x \to 0} \dfrac{x - \tan x}{x^3}$.

5) Find the n^{th} derivatives of $x^3 e^{ax}$.

6) Find the n^{th} derivatives of $\sin^3 x$.

7) If $y = e^{m \sin^{-1} x}$, prove that $(1 - x^2)y_2 - xy_1 = m^2 y$.

8) If $x = r\cos\theta$ and $y = r\sin\theta$, then $\left(\dfrac{\partial r}{\partial x}\right)^2 + \left(\dfrac{\partial r}{\partial y}\right)^2 = 1$.

9) Find the total derivatives of u w.r.t. 't' when $u = e^x$, where $x = \log t$, $y = t$.

10) If $u = x + y + z$, $v = y + z$, $w = z$, show that $\dfrac{\partial(u, v, w)}{\partial(x, y, z)} = 1$.

PART – B

Answer **any five** of the following questions. **(5×12=60)**

II. 11) State and prove Lagrange's mean value theorem.

12) Expand $\log(1 + x)$ by Maclaurins expansion.

P.T.O.

III. 13) Evaluate :

a) $\lim_{x \to \pi/2} (\sec x - \tan x)$.

b) $\lim_{x \to 0} \left(\frac{\tan x - x}{x^2 \tan x} \right)$.

14) Evaluate :

a) $\lim_{x \to 0} \frac{\log(\theta - \pi/2)}{\tan \theta}$.

b) $\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x}$.

IV. 15) Find the n^{th} derivatives of (a) $e^{ax} \cos(bx + c)$ (b) $y = \log(x^2 - 4)$.

16) Find the n^{th} derivative of $\frac{1}{x^2 + a^2}$.

V. 17) State and prove Leibnitz theorem on derivatives of product of two function.

18) Find the n^{th} derivatives of $x^2 e^x \sin x$.

VI. 19) If $y = e^{m\cos^{-1}x}$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

20) State and prove Euler's theorem on homogeneous function.

VII. 21) If $u = \sec^{-1}\left(\frac{x^3 + y^3}{x + y} \right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.

22) Find $\frac{dz}{dt}$, if $z = \log(x^2 - y^2)$ where $x = e^t \cos t$, $y = e^t \sin t$.

VIII. 23) If u, v are functions of r, s and r, s are functions of x, y, then prove that $\frac{\partial(u, v)}{\partial(r, s)} \cdot \frac{\partial(r, s)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(x, y)}$.

24) Find the Jacobian if $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$. Show that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = \frac{1}{4}$.

31228

NEP

B.Sc. II Semester Degree Examination, Sept./Oct. 2022

MATHEMATICS

Algebra – II and Calculus – II

Time : 3 Hours Max. Marks : 60

Instructions : *1) Part – **A** : **All** questions are **compulsory**.*

*2) Part – **B** : Answer **any five full** question.*

PART – A

1. Answer **all** the questions. **(10×1=10)**
   a) Define supremum.
   b) Define neighbourhood of a point.
   c) State Bolzano-Weierstrass theorem.
   d) Define Abelian group.
   e) Define sub-group.
   f) Define homogeneous function.
   g) State Euler's theorem on homogeneous function.
   h) Write Jacobian of u and v with respect to x and y.
   i) Define line integral along three dimensional space curve.
   j) Write equation of parabola and ellipse in Cartesian form.

PART – B

Answer **any five full** questions. **(5×10=50)**

2. a) Show that every infinite set has a denumerable sub-set.
   b) If x and y are two given real numbers with $x > 0$ then prove that there exists a natural number n such that $nx > y$.

P.T.O.

3. a) Prove that the intersection of two open sets is open.

   b) Show that, a non-empty set H of a group G is a sub-group of G iff a, b $\in$ H $\Rightarrow ab^{-1} \in H$.

4. a) Show that the multiplicative group of cube roots of unity is cyclic.

   b) State and prove Fermat's theorem.

5. a) If u = f(x, y) is a homogeneous function of degree n then prove that :

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu\cdot$$

   b) Find du if $u = x^2y + y^2z + z^2x$.

6. a) If $u = z - x$, $v = y - z$ and $w = x + y + z$, find the Jacobian $J=\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

   b) Obtain the Taylor's series expansion of f(x, y) = sin2x + cosy about a point (0, 0).

7. a) Evaluate $\int[(2y+x^2)dx+(3x-y)dy]$ along the curve x = 2t, $y = t^2 + 3$ where $0\leq t\leq 1$.

   b) Evaluate $\int_0^1\int_0^{x^2}(x^2+y^2)\,dydx$.

8. a) Evaluate $\int_0^1\int_0^x\sqrt{x^2+y^2}\,dy\,dx$ by changing into polar co-ordinates.

   b) Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dzdydx$.

11223

B.Sc. II Semester Degree Examination, September/October 2022
Paper – 2.1 : MATHEMATICS
Algebra – II (Old)

Time : 3 Hours | Max. Marks : 60

Instructions *: 1) Answer the questions* ***Section*** *wise.*

2) Write the question numbers ***correctly****.*

SECTION – A

I. Answer **any ten** of the following questions : **(10×2=20)**

1) State Euclid's algorithm theorem.
2) Using Descartes rules of sign find the number of positive and negative roots of $2x^7 - x^4 + 4x^3 - 5 = 0$.
3) Diminish the roots of the equation $2x^5 - x^3 + 10x - 8 = 0$ by 5.
4) Increase the roots of the equation $x^4 - 24x^2 - 13x + 35 = 0$ by 2.
5) Define divergence of sequence and convergence of a sequence.
6) Show that sequence $\left\{1-\frac{1}{n}\right\}$ is a monotonic increasing sequence.
7) Test the convergence of the sequence whose n^{th} term is given $1+ (-1)^{2n}$.
8) Find the limit superior and limit inferior for the sequence $\{x_n\} = \{(-1)^n\}$.
9) Test the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$.
10) Test the series $1^3 + 2^3 + 3^3 + \ldots + n^3$ for divergence.
11) State D' Alembert's ratio test.
12) State the Leibnitz theorem for an alternating series.

SECTION – B

II. Answer **any two** of the following questions : **(2×5=10)**

13) Solve the equation $x^3 - 18x - 35 = 0$ by Cardon's method.
14) Show that the equation $2x^7 + 3x^4 + 3x + K = 0$ has at least four imaginary roots for all values of K.

P.T.O.

15) Transform the equation $2x^3 - 9x^2 + 13x - 6 = 0$ into one in which the second term is missing and hence solve the equation.

16) Solve the equation $x^3 - 18x - 35 = 0$ by Cardon's method.

SECTION – C

III. Answer **any three** of the following questions : **(3×5=15)**

17) Prove that $\lim_{n\to\infty} (x_n.y_n) = l.m$ where $\{x_n\}$ and $\{y_n\}$ are sequence converging to l, m respectively.

18) Prove that a monotonic increasing sequence which is not bounded above diverges to $+\infty$.

19) Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.

20) Find the limit of following sequence 0.5, 0.55, 0.555, . . .

21) Show that the sequence $\{x_n\}$ defined by $x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$ is convergent and $2 \le \text{limit } x_n \le 3$.

SECTION – D

IV. Answer **any three** of the following questions : **(3×5=15)**

22) State and prove P-series.

23) State and prove Cauchy's n^{th} root test.

24) Discuss the convergence of the series :

i) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$

ii) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots$

25) Discuss the convergence

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \ldots + \frac{x^n}{n^2+1}.$$

26) Test the convergence of

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots$$

11224

B.Sc. II Semester Degree Examination, September/October 2022
MATHEMATICS
Paper – 2.2 : Calculus – II (Old)

Time : 3 Hours   Max. Marks : 60

Instructions : *1) Answer **all** the Sections.*
*2) Mention the question number **correctly**.*

SECTION – A

I. Answer **any ten** of the following questions. **(10×2=20)**

1) Show that $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$.

2) Integrate $\frac{\sec^2 x}{\tan^2 x - 4}$ w.r.t.x.

3) Evaluate $\int \frac{dx}{5+4\cos x}$.

4) Integrate $\frac{2x-3}{\sqrt{2x^2-7x+5}}$ w.r.t.x.

5) Evaluate $\int \frac{xe^x}{(1+x)^2}\,dx$.

6) Evaluate $\int x\sin 3x\,dx$.

7) Evaluate $\int \frac{x-1}{(x-2)(x-3)}\,dx$.

8) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\,dx$.

9) Evaluate $\int_0^1 e^x \, dx$ as limit of a sum.

10) Evaluate $\int_0^{\pi/2} \cos^6 x \, dx$.

11) If u = xtany + ytanx, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

12) If u = 3x + 5y, v = 4x – 3y, show that $\frac{\partial(u, v)}{\partial(x, y)} = -29$.

SECTION – B

II. Answer **any five** of the following questions. **(5×5=25)**

13) Evaluate $\int \frac{2x-3}{x^2+3x-18} dx$.

14) Evaluate $\int \frac{dx}{5+7\cos x+\sin x}$.

15) Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

16) Evaluate $\int_0^2 (x+4) \, dx$ as a limit of a sum.

17) Find the reduction formula for $\int \sin^n x dx$, where n is the integer hence evaluate $\int_0^{\pi/2} \sin^n x dx$.

18) Find the reduction formula for $\int \sec^n x dx$.

19) Find the area of the curve Asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

SECTION – C

III. Answer **any three** of the following questions. **(3×5=15)**

20) If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, S.T $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

21) State and prove Euler's theorem for homogeneous functions.

22) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\phi$.

23) If u and v are functions of two independent variables s and t. And s and t themselves are functions of two independent variables x and y, then $\frac{\partial(u, v)}{\partial(s, t)} \cdot \frac{\partial(s, t)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(x, y)}$.

24) Show that the functions

$u = e^x \log y + xyz$

$v = \log x + e^y + xyz$

$w = xyz$ are not functionally dependent.

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27223(New)

B.Sc. II Semester Degree Examination, September/October - 2021

MATHEMATICS

Calculus - I

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates : Part A : All questions are compulsory.

Part B : Solve any five from seven question (Each question carries equal marks)

Part - A

I. Answer **all** the following questions. **(10×2=20)**

1. State Roll's theorem and show that it is in applicable for $f(x) = x^3$ in [1,2].
2. Verify Lagrange's mean value theorem for $f(x) = x^2 - 3x - 2$ in [-2,3].
3. Expand $f(x) = e^{-x}$ up to the terms containing x^5 using Maclaurins series.
4. Evaluate $\lim_{x \to 0} \frac{e^x + e^{-x} - 2x}{x^2 \sin x}$.
5. Find the n^{th} derivative of $x^3 e^{ax}$.
6. Find the n^{th} derivative of a^{mx}.
7. If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_2 - xy_1 = m^2 y$.
8. If $u = \phi(y+ax) + \psi(y-ax)$. Show that $\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}$.
9. Find the total derivative u w.r.t. 't' when $u = e^x$ where x = -logt, y = t.
10. If $u = x+y+z, v = y+z, w = z$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 1$.

Part - B

Answer any **five** of the following. **(5×12=60)**

II. 11. State and prove Cauchy's mean value theorem.

12. Expand $\log(1+x)$ by maclaurin's expansion.

[P.T.O.

III. 13. Evaluate

a. $\lim_{x\to\frac{\pi}{2}}(\sec x - \tan x)$.

b. $\lim_{x\to 0}\left(\frac{\tan x - x}{x^2 \tan x}\right)$.

14. Evaluate

a. $\lim_{x\to 1} it\left(\frac{\sqrt{x}-1+\sqrt{x+1}}{\sqrt{x^2-1}}\right)$.

b. $\lim_{x\to 0} it\left(\frac{\tan x}{x}\right)^{1/x}$.

IV. 15. Find the n^{th} derivative of

i. $e^{ax}\sin(bx+c)$

ii. $e^x \sin x \cos 2x$.

16. Find the n^{th} derivative of $\frac{1}{x^2+a^2}$.

V. 17. State an prove Leibnitz theorem on derivative of product of two function.

18. Find the n^{th} derivative of $x^2 e^x \sin x$.

VI. 19. If $y = e^{m\cos^{-1}x}$. Show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

20. State and prove Euler's theorem on Homogeneous function.

VII. 21. If $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.

22. If $w = f(x,y)$, $x = r\cos\theta, y = r\sin\theta$. Show that $\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$.

VIII. 23. If u, v are functions of r, s and r,s are function of x,y, then prove that $\frac{\partial(u,v)}{\partial(r,s)}\cdot\frac{\partial(r,s)}{\partial(u,v)} = \frac{\partial(u,v)}{\partial(x,y)}$

24. Find the Jacobian if $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = \frac{1}{4}$.

9897589

27223(New)

B.Sc. II Semester Degree Examination, September - 2020

MATHEMATICS

Calculus-I

(New)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates: Part-A- All questions are compulsory.
Part-B-Solve any five from seven questions

PART-A

I. Answer all the following. **(10×2=20)**

1. Verify Rolle's theorem for the function f (x)= x^3–4x in [–2,2]

2. Verify Lagrange's mean value theorem for f (x) = (x–1)(x–2)(x–3) in [0,4].

3. Expand the function f (x) = sinx up to the term containing x^5 by Maclaurin's theorem.

4. Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

5. Find the n^{th} derivative of cos5x cos3x

6. If y=acos(log x) + bsin(logx) show that $x^2y_2+xy_1+y=0$

7. Find the n^{th} derivative of $\frac{2x-1}{(x+1)(x-2)}$

8. If $u = \tan^{-1}\frac{y}{x}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

9. Find $\frac{dz}{dt}$ if z = x^2+y^2 Where x=e^t cost, y=e^t sint.

10. If x+y=u and y=uv

    find $\frac{\partial(u,v)}{\partial(x,y)}$

[P.T.O.

PART-B

(5×12=60)

Answer any Five of the following.

II. 11. State and prove Lagrange's mean value theorem

12. Expand the function log (1+sinx) upto the term containing x^3 by Maclaurin theorem and deduce $\log(\sec x) = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \text{----}$

III. 13. Evaluate

a) $\lim_{x \to 0} (\cos x)^{1/x^2}$

b) $\lim_{x \to 0} \frac{\log \tan 2x}{\log \tan x}$

14. Evaluate

a) $\lim_{x \to b} \frac{x^b - b^x}{x^x - b^b}$

b) $\lim_{x \to 0} x \log x$

IV 15. Find the n^{th} derivative of $e^{ax} \sin (bx+c)$

16. Find the n^{th} derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

V 17. Find the n^{th} derivative of $x^3 \sin ax$

18. If $y = x^n \log x$ show that $y_{n+1} = \frac{n!}{x}$

VI 19. If $y = \sin (m \sin^{-1} x)$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$

20. Verify Euler's theorem for the function

a) $u = x^3 \log\left(\frac{x}{y}\right)$

b) $u = \frac{x^{1/3} - y^{1/3}}{x + y}$

VII. 21. If u=f (y–z, z–x, x–y) Show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

22. If $w=f(x,y)$, $x=r\cos\theta$, $y=r\sin\theta$, show that

$$\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2$$

VIII 23. If u=f (x,y) and v=g(x,y) then prove $\frac{\partial(u,v)}{\partial(x,y)}\cdot\frac{\partial(x,y)}{\partial(u,v)}=1$

24. if $x=u+v$, $y=w+v$, $z=w+u$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{1}{2}$

B.Sc. II Semester Degree Examination, April/May. - 2019

MATHEMATICS

Calculus-I

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

Part A : **All** questions are Compulsory.

Part B: Solve any **Five** from Seven questions.

PART - A

Answer **ALL** the following. **(10×2=20)**

1. State Rolle's theorem and show that it is inapplicable for $f(x)=x^3$ in $[1, 2]$.

2. Verify Lagrange's mean value theorem for $f(x)=(x^2-4)^{1/2}$ in [2,3.]

3. Expand $f(x)=e^{-x}$ up to the terms containing x^5 using Maclaurins Series.

4. Evaluate $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$

5. If $y=x^m$, where m is a positive integer then show that $y_n=\frac{m!}{(m-n)!}x^{m-n}$. Also discuss the possibilities when n = m and n > m.

6. Find the n[th] derivative of $y=\sin^3 2x$

7. If $y=e^{m\cos^{-1}x}$ prove that $(1-x^2)y_2-xy_1=m^2y$

8. If $x=r\cos\theta$ and $y=r\sin\theta$ then $\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2=1$

[P.T.O.

9. Find the total derivative of $u = x^2 - y^2$ where $x = e^t \cos t$, and $y = e^t \sin t$.

10. If $u = x(1+y)$ & $v = y(1+x)$ show that $\frac{\partial(u,v)}{\partial(x,y)} = 1+x+y$.

PART - B

(5×12=6

Answer any **FIVE** of the following.

II. 11. State and prove Cauchy's mean value theorem.

12. Obtain the Maclaurin's expansion of $\log(1+x)$ and hence dedu

$$\log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots$$

III. 13. Evaluate a) $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

b) $\lim_{x \to \infty}\left(a^{1/x} - 1\right)x$

14. Evaluate a) $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x}$

b) $\lim_{x \to 0}\left(\frac{1}{x}\right)^{2\sin x}$

IV. 15. Find the nth derivative of $y = e^{ax}\cos(bx+c)$

16. If $y = \frac{x^4}{x^2 - 3x + 2}$ then find y_n.

V. 17. State and prove Leibnitz theorem on derivative of product of two functions.

18. Find the nth derivative of $\cos^2 x.\sin^4 x$

VI. 19. If $y=e^{a\cos^{-1}x}$ show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$

20. State and prove Euler's theorem on homogeneous functions.

VII. 21. Verify Euler's theorem for $u=\sin\left(\frac{y}{x}\right)+\cot^{-1}\left(\frac{x}{y}\right)$

22. If $u=\tan^{-1}\left(\frac{x}{y}\right)$ where $x^2+y^2=4$. Show that $\frac{du}{dx}=\frac{1}{\sqrt{4-x^2}}$.

VIII.23. If u, v are functions of r, s and r, s are functions of x, y then $\frac{\partial(u,v)}{\partial(r,s)}\frac{\partial(r,s)}{\partial(x,y)}=\frac{\partial(u,v)}{\partial(x,y)}$

24. If $u=\frac{x_2x_3}{x_1}$, $v=\frac{x_3x_1}{x_2}$, & $w=\frac{x_1x_2}{x_3}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$.
